STUDENTS' INITIAL UNDERSTANDING OF THE CONCEPT OF CONSERVATION OF AREA

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Abstract

This study answers how the students understand the initial idea (recomposing shape) of the concept of conservation of area. The instructional activity which was made in this study is based on realistic mathematics education. The activity was created for third grade elementary school students in SD Al-Hikmah Surabaya. The third grade students were used since they have not learned about area measurement. So they could focus just on the shape without disturbance of the area formula. The result of this study suggest that students still need more similar activities to ensure their understanding toward the idea that recomposing shape will preserve the area of the shape.

Keywords: Concept of Conservation of Area, Initial Understanding, Recomposing Shape

Abstrak

Penelitian ini menjawab bagaimana siswa memahami ide awal (*recomposing shape*) dari konsep konservasi luas. Aktivitas instruksional yang dibuat dalam penelitian ini berdasarkan Realistic Mathematics Education (RME). Aktivitas ini dibuat untuk siswa SD Al-Hikmah Surabaya kelas 3. Siswa kelas 3 dirasa cocok karena mereka belum belajar tentang konsep pengukuran luas. Sehingga mereka bisa lebih fokus kepada bentuk bangun datarnya tanpa diganggu oleh rumus luas. Hasil dari penelitian ini menyarankan bahwa siswa masih butuh beberapa aktivitas serupa untuk memastikan pemahaman mereka bahwa *recomposing shape* mengawetkan luas dari suatu bangun.

Kata kunci: Konsep Konservasi Luas, Ide Awal, *Recomposing Shape*

The concept of conservation of area is preliminary step in students' adequate mastering of area measurement (Kospentaris, Spyrou & lappas, 2011). This concept is suggested to be mastered by children in the concrete operation (7-12 years old) of the Piaget's cognitive development. Also, Piaget argued that understanding the concept of conservation of area is necessary and prerequisite to students understanding the concept of area measurement as well as multiplication structure (Kordaki, 2003). Kordaki (2003) conducted a study to investigate the strategies developed by 14-years old students regarding the concept of conservation of area. Meanwhile Kospentaris, Spyrou & lappas (2011) also investigate the strategies employed on the concept of conservation of area by advanced high school and university students. Since previous research concerning area conservation investigated performance of the junior, senior and even university students, it would of interest to assess deeper the way of the concept of conservation of area being understood by the children of 9-10 years old.

In this study, we designed a contextual problem based on RME (Realistic Mathematics Education) in order to give the students opportunity to infer from immediate experience rather than

directly perceived. The context making table-cloth is chosen considering the culture of Indonesian students as the subject of this study. Moreover, we studied the process of the students in understanding the concept of conservation of area not only the result or the students' answer. Because it is the first time for the students to learn the concept of conservation of area, the initial understanding of the students appears as an aspect of the research with the special interest.

The term of "conservation" means the invariance of the quantitative value of the area of a figure while the figure may be transformed into a qualitatively different one (Piaget, Inhelder &Szeminka, 1960 as cited in Kospentaris, Spyrou & lappas, 2011). Understanding that re-arrangement of the parts of a figure does not alter the areas of the figure have been considered a prerequisite to the concept of conservation of area (Kospentaris, Spyrou & lappas, 2011). As well as the notion of changing only its position and splitting into some parts, recomposing the figure to produce an equivalent figure needs to be learned by the students (Kordaki, 2003). In the same way, the notions of reversibility and transitivity also become fundamental aspect to learn the concept of conservation of area (Piaget et al., 1960 as cited in Kospentaris, Spyrou & lappas, 2011). Children will not easy to understand that recomposing a figure will preserve the area of the figure. As they still have difficulties in understanding the possibility of equivalence of an area when it is represented in figures of different forms (Carpenter, et al, 1975 as cited in Kordaki, 2003). In order to make the children understand that recomposing a figure will preserve the area, the children need to understand the notion of reversibility and transitivity. Therefore, the notion of reversibility, transitivity and recomposing figures will be the initial concept to underlie the children understanding into the concept of conservation of area.

Children in all over the world are firstly taught the concept of area measurement in their Primary School. As Piaget argued that the concept of conservation of area should be taught as a preparatory for learning the area measurement, so the concept of conservation of area in this study will be taught for the third grader (9-10 years-old) students in the Primary School in Indonesia.

Meanwhile, in the Indonesian curriculum the concept of conservation of area do not explicitly stated. As well Clement &Stephan (2004 as cited in Kospentaris, Spyrou & lappas, 2011) emphasized that the concept of conservation of area is often neglected in instruction. In addition, Kordaki (2003) stressed that in the school the concept of conservation of area is overlooked and isolated from the concept of area measurement and area formula. Hence, in our study the third grader students will be learned the concept of conservation of area before they learn the area measurement.

In this study, we used RME approach to provide a contextual embedded activity in designing the learning instructional activities to teach the children about the concept of conservation of area. The context making table cloth is chosen since it could raise the students' awareness of the idea of recomposing a figure. As well as the principle of RME which is guided reinvention, the context making table-cloth is hoped to be able to encourage and guide the students to reinvent the need and the idea of conservation of area.

The instructional activities that made should follow the five tenets of RME which defined by Treffers in Bakker (2004). The following description explains how these tenets are adopted in this study.

1. Phenomenological exploration

   A concrete meaningful context, 'making table cloth from the cloth-rag", is elaborated in this study to bring the students to use the strategy cut and paste for making the table cloth.
2. Using models and symbols for progressive mathematization

   The way to determine which part should be cut and where to place will be used as a model to facilitate the students' progressive mathematization, from the intuitive into more formal mathematical concepts.
3. Using students' own constructions and productions

   The students will produce their own way when cutting and pasting the shape as the meaningful strategy for them.
4. Interactivity

   There will be a class discussion and working group in which students could communicate, compare and justify their ideas with each other.
5. Intertwinement

   The design of this study will connect the notion of conservation of area with the concept of area measurement.

The way of the students get their initial understanding of the concept of conservation of area by using the "making table-cloth" activity is considerably interest. All the consideration led us to the following research question:

"*How do the students understand the initial idea (recomposing shape) of the concept of conservation of area?"*

METHOD

This study used design research as the method. There were two cycles used in this study, Cycle 1 and Cycle 2. Cycle 1 will tested the HLT (Hypothetical Learning Trajectory) which made as the preparation of this instructional activity then the result of Cycle 1 will be used to revise the HLT. The revise HLT will be implemented in Cycle 2 as the focus of this study. The learning process in the class will be recorded to gain the accurate data. The written work of the students also supported the triangulation of the data. The subject of this study was the third grader of the Moslem private Primary School (SD AL-Hikmah) in Surabaya, Indonesia. Cycle 1 conducted with six students with heterogenic academic level of class 3C by the researcher as the teacher. Meanwhile, Cycle 2 conducted with all students in the class 3A by Bu Lila as their mathematics teacher.

The making table-cloth activity is aimed to arouse the students' awareness of the concept of conservation of area. The cloth-rag is chosen because this context can give the students natural situation to cut and paste the cloth. Also, the cloth-rag is usually formed in the non-regular shape due to the rest of making cloth. The students will use the idea of recomposing the cloth-rag to fit the table. They will also understand the effect of recomposing shape (after recomposing the cloth-rag could fit the table, so cloth-rag and table have the same area) and the idea of reversibility (the area of cloth-rag before and after cutting remains the same).

The individual worksheet (worksheet 1) will asked the students to imagine whether a cloth-rag which shown in the paper could cover the table which also shown in the paper perfectly or not. The cloth-rag is twice longer but a half wider than the table. This time, the students still use their perceptual justification. Later on in Worksheet 2, the students were divided into group consisted of 4-5 students to demonstrate their perception. The students were asked to prove their answer by giving the model of the cloth-rag and the Barbie's table. There will also some questions that ask the students to choose which one is bigger the cloth-rag or the table. They are expected to get the conclusion of the recomposing shape and reversibility in Worksheet 2. The following picture is shown the cloth-rag and the table which is shown to the students in Worksheet 1.

Figure 1. The picture of the cloth-rag and the table of Worksheet 1

RESULTS AND DISCUSSION

Retrospective Analysis of Cycle 1

In the cycle 1, the students easily understood the question in Worksheet 1 since all of them have been familiar with the context. In the introduction of the context they said that they knew what is a tailor, what is her job, what is cloth-rag, what do people usually use the cloth-rag for and etc.

In Worksheet 1, the students have to imagine the possibility to make a table-cloth from a cloth-rag which has twice longer and a half width of the size of the table. The table and the cloth-rag were shown in Worksheet 1. Since the story in the Worksheet 1 said that Bu Tina has many cloth-rags, there were some students who think that they can use the other cloth-rag which are not shown in the

worksheet. Actually the story of Problem 1 has been explained that Bu Tina now just has the cloth-rag showing in the worksheet for making the table-cloth.

All of the students in Cycle 1 answered that the cloth-rag can be made for a table-cloth with different reasons. Mostly students argued that Bu Tina still has many cloth-rags to be used for covering the table precisely. But there was a student who answered that the cloth-rag can be cut and can be combined to cover the table. Hence, the idea to do recomposing shape by cut and paste strategy was revealed from the students themselves in the making table-cloth activity.

Actually the story is seemed too long for being understood by the students. Also, they were too depended on the teacher' explanation for understanding the problem. Therefore, when the students read the story, *"Because Bu Tina is a tailor, she had many cloth-rag in her house."*, they paid less attention into the next sentence which is, *"But Bu Tina now just has cloth-rag that twice longer and a half wider than the table."*. So they thought that Bu Tina has many cloth-rags to make table-cloth. Hence, we can delete the story that Bu Tina has many cloth-rags so the students will be directed to the fact that the only cloth-rag is showed in the worksheet. We will emphasize that Bu Tina now just has one and only one cloth-rag which is shown in the worksheet. Moreover, the students' capability to understand the concept of half of the width of the cloth rag which is a half of the table was not discovered yet. It happened because none of the students talked about the measurement of the cloth-rag of the table in Worksheet 1.

Worksheet 2 is used for proving the students' answer in the Worksheet 1. Thus, the students were given the fabric of the cloth-rag with the size twice longer and a half wider than the table. The table used in Cycle 1 was the Barbie's table. The students could use these tools as visual aids to answer the questions in the worksheet. Problem 1 asked them to prove whether it is possible to make a table cloth from the cloth-rag, the students can easily find the way to make the table cloth. They asked whether it is okay if they cut the cloth. Then they placed the cloth-rag horizontally in the table and found where to cut (see Figure 1.2 no 1). From this activity, the students knew that they have to cut the cloth-rag into two equal parts (see Figure 2. no 2). Next, the students combined these two parts together and sewed on the table (see Figure 2. no 3).

Figure 2. The process of students answer (superposition, know where to cut, and cover the table with the cloth-rag)

The students' conclusion stopped on their hands-on activity after the cloth-rag can be made for the table-cloth. But when they were asked which one is bigger, they were confused. Generally, the students were confused with the word bigger and the size. They thought that big is related with size, then when you cut the cloth the size changes, the length and the width change. So, they thought that the table is bigger than the cloth-rag which has been cut into smaller parts.

Therefore, the teacher asked the students whether the cloth-rag is same or not before and after cutting. Some of the students answered same, some answered not. They argued the cutting process that makes the cloth-rag before and after cutting are not the same. Overall, the students still could not understand the idea that the cloth-rag will have the same area as the table if and only if they cover each other precisely.

Revised HLT

In Worksheet 1, the story that Bu Tina has many cloth-rags in her house was deleted because many students answer that it was possible to make the table cloth since Bu Tina still has many cloth-rags. Meanwhile, actually the story has told the students that now Bu Tina just has one cloth-rag like shown in the worksheet.For problem number 2 in Worksheet 2, it should provide the alternative

answer like, *"Compare the cloth-rag and the table, which one is bigger? The table, the cloth-rag or both of them are the"*. It should be done as when the question asked which one is bigger, the students thought they have to choose, either A or B. They were often to think that both of them can be same big or none of both is bigger than each other. Moreover, when look from the meaning of the question, it was less sufficient to lead the students to answer like our expectation.

Retrospective Analysis of Cycle 2

When Worksheet 1 was given to the students, the teacher emphasized that the cloth-rag in Worksheet 1 was just one and only one. So the students could not think that Bu Tina has another cloth-rag in her house. Then the teacher discussed with the students the meaning of a half. The students seemed to know what is a half, but they could not communicate it. Therefore the teacher used an illustration as the following, *"If the width of the paper is 10 cm, so what is the half of it?"*. The students answered correctly which is 5 cm. From this activity we could see that the students have known the meaning of half. The teacher also gave comprehension to the students about the meaning of twice longer by the illustration of paper and using measurement (cm). The role of the teacher in Worksheet 1 determined the students' understanding of the problems. When the students understood the problems, then it will lead the students to answer as the expectation.

Furthermore, some students could think of the strategy to cut and paste the cloth-rag to fit the table. The modified story in Worksheet 1 made the students to directly go to the problem, not too much attention to the story. Most of the students answered that it is possible to make the table cloth from the given cloth-rag because the cloth-rag can be cut and be arranged to fit the table. For instance, the cloth-rag should be measured to the table then cut the leftover then sewed it beside the previous cloth-rag.

All the students in the focus groups answered that the cloth-rag could be used to make the table cloth but they had different argument. Some of them argued about the size of the cloth-rag which is longer than the table but also a half wider than the table, so it could be fit. Even, Iman have thought to cut the cloth-rag into two and to sew the top and the bottom of the parts. We believed that Iman thought about the correct answer, but he might be not able to communicate his thinking clearly in his answer.

From the students' understanding about twice and half, they thought to cut the cloth-rag in a half then combined the two equal parts to cover the table. But there were a few students who thought that it was impossible to make the table cloth since the table is wider. However, these students have changed their mind after they did Worksheet 2 later on.

Therefore, we could say that comparing the shapes, thinking to cut the cloth-rag and to paste to the table and even planning to sew the pieces of the cloth-rag to fit the table, was the starting point to understand the concept of conservation of area for the students.

In Worksheet 2, the students in the focus group have known that the cloth-rag can fit the table from their experiment, but in the group discussion they still tent to focus on the dominant dimension.

Also, the focus group answered that '*the table is wider while the cloth-rag is longer*'. But the focus group also wrote that dividing the cloth-rag into two could make the width same with the table. So we could see that the students still confused with their experiment. They knew that recomposing cloth-rag could fit the table, but they still thought that the cloth-rag is bigger than the table because it was longer than the table. Hence, it shows that the students still confused with the size of the shapes.

Therefore, in Cycle 2, we gave several additional activities which called inter-perception in order to strengthen the new concept they have just had. The **inter-perception** aimed to strengthen the students understanding about the concept of reversibility in the conservation of area. In the inter-perception , the students were showed a figure of three cloth-rags in the LCD. The first cloth-rag was the cloth-rag before cutting. Then the cloth-rag was cut into two equal pieces and separated in the middle to be numbered as the second cloth-rag (see Figure 3.). The third cloth-rag was the combined cloth-rag to be fit with the table.

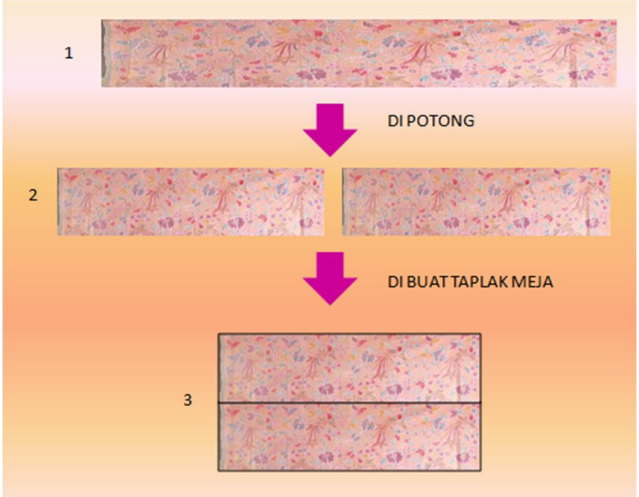

Figure 3. Inter-perception 1 for understanding the concept of reversibility

From the figure that showed in the LCD, the teacher asked to the students, "*Which one is bigger, the cloth-rag in number 1 or in number 3? Why?"*. The intention of the question was to ask whether the cloth-rag in number 1 and number 3 has the same area or not. We could not use the term 'area' in this learning process, so we used the question "*which one is bigger"*.

At the first, the students had different answer. Some of them said that the area of the cloth-rag before and after cutting was not the same, some said the same. They looked confuse and not confident with their own answer. In order to check the students' answer, the teacher repeated the question for several time. But the class discussion still sounded off two answers, the same and not the same. Therefore, the teacher gave some clues, such as, *"Is there any part of the cloth-rag that being wasted? or is there any part that added in the cloth-rag?"*. The students answered, "*No*". From this clue, the students could get some insight about the fact that the cutting of the cloth-rag will preserve the area as long there are not parts that wasted.

In the end of the inter-perception 1, students have been able to conclude that although two shapes have different form, but it is possible to have the same area. From this activity, we could see that the role of the teacher was very important to lead the students understand the purpose of the discussion. However, it was the last hour of the day, so the students were not fully concentrated. They looked agree with the result of the discussion but it can be seen from their face that some of the students were still confused.

CONCLUSION AND SUGGESTION

At the first it seemed not easy for the students to understand the recomposing idea, especially in the first cycle. But in the second cycle when the role of the teacher was emphasized and with the help of inter-perception the students could develop their understanding that recomposing shape will preserve its area. From the first and the second cycle we noted that the students could understand the initial idea of the concept of conservation of area starts from the effect of recomposing shape and reversibility (returning back into original). The effect of recomposing shape which say that the cloth-rag could cover table precisely, make the students think about the possibilities of cloth-rag and table have the same area. Then, by the reversibility that emphasized in inter-perception, which asked the students whether the cloth-rag after and before recomposing have the same area, make the students aware that the cloth-rag that they cut and combine come from the same cloth-rag. So it must have the same area since when they return back the tablecloth could fit the cloth-rag.

Furthermore, by looking the struggle of the students to understand the recomposing idea means that they still need more similar activities to strengthen their understanding. We suggest to give further activity like comparing two different shape which actually have the same area. When they face this problem, they will try to trace the two shapes and find which one is prominent of one shape and which one is protrudes of another shape. They will try to put the prominent into the protrudes and they will see that it fit each other. So this activity could strengthen their understanding that recomposing shape will preserve its area although they have different form.

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