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Developing a Model to Support Students in Solving Subtraction

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Abstract

Subtraction has two meanings and each meaning leads to the different strategies. The meaning of "taking away something" suggests a direct subtraction, while the meaning of "determining the difference between two numbers" is more likely to be modeled as indirect addition. Many prior researches found that the second meaning and second strategy rarely appeared in the mathematical textbooks and teacher explanations, including in Indonesia. Therefore, this study was conducted to contribute to the development of a local instruction theory for subtraction by designing instructional activities that can facilitate first grade of primary school students to develop a model in solving two digit numbers subtraction. Consequently, design research was chosen as an appropriate approach for achieving the research aim and Realistic Mathematics Education (RME) was used as a guide to design the lesson. This study involved 6 students in the pilot experiment, 31 students in the teaching experiment, and a first grade teacher of SDN 179 Palembang. The result of this study shows that the beads string could bridge students from the contextual problems (taking ginger candies and making grains bracelets) to the use of the empty number line. It also shows that the empty number line could promote students to use different strategies (direct subtraction, indirect addition, and indirect subtraction) in solving subtraction problems. Based on these findings, it is recommended to apply RME in the teaching learning process to make it more meaningful for students.

Keywords: Subtraction, Design Research, Realistic Mathematics Education, The Beads String, The Empty Number Line

Abstrak

Pengurangan memiliki dua arti dan masing-masing arti mengarahkan ke strategi yang berbeda. Arti pengurangan sebagai "mengambil sesuatu" mendukung pengurangan langsung, sedangkan arti pengurangan sebagai "menentukan perbedaan dari dua bilangan" lebih mudah dimodelkan sebagai penjumlahan tidak langsung. Banyak penelitian sebelumnya menemukan bahwa arti pengurangan yang kedua dan strategi yang kedua jarang muncul di dalam buku matematika dan penjelasan guru, termasuk di Indonesia. Oleh karena itu, penelitian ini dilaksanakan dalam rangka memberikan kontribusi bagi pengembangan local instruction theory untuk pengurangan dengan mendesain aktivitas pembelajaran yang dapat memfasilitasi siswa kelas 1 sekolah dasar untuk mengembangan model dalam menyelesaikan pengurangan bilangan dua angka. Konsekuensinya, design research dipilih sebagai pendekatan yang sesuai untuk mencapai tujuan penelitian dan Realistic

Mathematics Education (RME) digunakan sebagai panduan untuk mendesain pembelajaran. Penelitian ini melibatkan 6 siswa dalam pilot experiment, 31 siswa dalam teaching experiment, dan seorang guru kelas satu SDN 179 Palembang. Hasil dari penelitian ini menunjukkan bahwa manik-manik dapat menjembatani siswa dari masalah kontekstual (mengambil permen jahe dan membuat gelang biji-bijian) ke penggunaan garis bilangan kosong. Ini juga menunjukkan bahwa garis bilangan kosong dapat mendorong siswa untuk menggunakan strategi yang berbeda (pengurangan langsung, penjumlahan tidak langsung, dan pengurangan tidak langsung) dalam menyelesaikan masalah pengurangan. Berdasarkan penemuan ini, direkomendasikan untuk mengaplikasikan RME dalam proses belajar mengajar agar membuat pembelajaran semakin bermakna bagi siswa.

Kata kunci: Pengurangan, Design Research, Realistic Mathematics Education, Manik - Manik, Garis Bilangan Kosong

Introduction

In solving subtraction problems, students have to think about the meaning of subtraction and the more efficient strategies to solve it. According to Fosnot and Dolk (2001), subtraction has two meanings; those are “taking away something” and “determining the difference between two numbers”. Each meaning leads to the different strategies. The context of “taking away something” suggests a direct subtraction. On the other hand, the context of “determining the difference between two numbers” is more likely to be modeled as indirect addition (Torbeyns, De Smedt, Stassens, Ghesquiere, & Verschaffel, 2009).

However, the indirect addition strategy, particularly with multi digit numbers, has received a little attention from researchers. The limited research interest for this complement strategy of direct subtraction is quite surprising because there are indications that indirect addition is not only computationally remarkably efficient but also very promising from a broader educational perspective (Torbeyns *et al.*, 2009).

Moreover, in the Indonesian mathematical text books (see Djaelani & Haryono, 2008), the meaning of subtraction is explained only as “taking away something”. Teachers provide only removal contexts in teaching subtraction. In a traditional teaching learning method, teachers also teach students an algorithm of subtraction directly, subtracting tens and ones separately, after they learn subtraction up to 20 by doing physical activities or using drawing. It is meaningless for students because they do this procedure without understanding (Kamii & Lewis, 1993).

In this situation, teachers need to emphasize that subtraction also has a meaning of "determining the difference between two numbers" that will be more efficient to solve by indirect addition. Therefore, this present study tries to provide a proper learning environment by designing a sequence of meaningful mathematical activities to promote students in constructing their understanding of the meaning of subtraction and in choosing the more efficient strategy to solve subtraction problems up to 100.

The aim of this study is to contribute to the development of a local instruction theory for subtraction by designing instructional activities that can facilitate students to develop a model in solving two digit numbers subtraction. Therefore, the central issue of this study is formulated into the following general research question: *How can a model support students to solve subtraction problems up to two digit numbers in the first grade of primary school?*

Theoretical Framework

Subtraction

According to Fosnot and Dolk (2001), subtraction has two meanings; those are "taking away something" and "determining the difference between two numbers". The first meaning mostly appeared in the mathematical textbooks and teacher explanations. In the first meaning, the only matching action is that of removing. Torbeyns, *et al.* (2009) described the strategies to solve subtraction problems in three different ways. They distinguished (1) direct subtraction, which means taking away the subtrahend from the minuend; (2) indirect addition, which means adding on from the subtrahend until the minuend is reached; and (3) indirect subtraction, which means subtracting from the minuend until the subtrahend is reached.

Subtraction in the Indonesian Curriculum

This present study will focus on subtraction in the second semester of grade 1. In this grade, students already had the sense of subtraction up to 20 by doing physical activities or using drawing in the first semester. At the other hand, in the beginning of second semester, they have not been taught the algorithm of subtracting tens and ones separately; and also borrowing and carrying procedures. It makes easier to develop a model and to build students number sense of subtraction.

The Empty Number Line in Solving Subtraction Problems

The powerful tool to support the "two ways traffic" of subtraction (taking away and adding on) is the empty number line. It was Freudenthal (1983) who pleaded for using what he called "geometrical concreteness of the number line" in which the two methods connected to the two interpretations of subtraction can be observed, namely "taking away at the start" and "taking away at the end". Studies conducted by Veltman in 1993 and Klein in 1998 demonstrated that the empty number line is a useful scheme for adding and subtracting up to 100 (Menne, 2001). In this study, a string of beads is used as a stepping stone in moving from contextual problems to the use of empty number line as a powerful model in solving subtraction problems up to two digit numbers.

Realistic Mathematics Education (RME)

The process of designing a sequence of instructional activities was consulted by five tenets for RME defined by Treffers (1987). Those tenets are described as following.

1. The use of contexts in phenomenological exploration

   The mathematical activity is started from local contexts situation that are experientially real for students. The context of taking ginger candies is used to construct the meaning of subtraction as "taking away something". For constructing the meaning of subtraction as "determining the difference between two numbers", the context of making bracelets from grains is used. Those contexts are familiar for Indonesian students.

2. The use of models for progressive mathematization

   Models are used as a bridge from concrete level to more formal level. First, students will explore the different contexts of taking candies and making bracelets. Then, they will make a visualization of the solution in their own way. Later on, a string of beads can serve as a powerful model to represent the situation of those contexts. Students can see the meaning of subtraction given in the beads string. In the next level, an empty number line can represent the general situation and can reflect the students' thinking.

3. The use of students' own constructions and productions

   In the activity of taking candies and making bracelets, students are given the opportunity to solve the problem in their own strategy. Class discussion is

conducted to discuss different ways to solve subtraction so that every student can get new insight from their friends and can choose more efficient strategy that makes sense for them.

4. The interactivity of the teaching process

   Teacher plays a role as a facilitator to support students' understanding by providing social interaction in the classroom. In solving subtraction problems, students always work in group and will share their idea to others. By this interaction, students can develop their thoughts and can learn to respect each other.

5. The intertwining of various mathematics strands or units

   The sequence of instructional activities in this study not only emphasizes the meaning of subtraction and the strategy to solve subtraction problems, but also stresses the relation between addition and subtraction. Moreover, students are not taught the algorithm of subtraction directly in order to build their number sense.

Pendidikan Matematika Realistik Indonesia (PMRI)

After ten years of PMRI development and pilots, a vast body of knowledge has been acquired on PMRI and on what is considered good PMRI education in Indonesia. Many experiences contributed to the slowly developed ideas of good standards for various aspects of PMRI, including PMRI lesson (Hadi, Zulkardi, & Hoogland, 2010).

Methodology

This study will be based on the design research approach as an appropriate methodology for achieving the research aim. Design research consists of three phases; those are preparing for the experiment, conducting the design experiment, and carrying out the retrospective analysis (Gravemeijer & Cobb, 2006).

This present study was conducted in the SDN 179 Palembang which is the partner school of PMRI. It contained two cycles of design experiment, namely pilot experiment and teaching experiment, and it took place on February until April 2012. The first cycle serves as a try out experiment in adjusting and improving the designed Hypothetical Learning Trajectory (HLT) to get the better design for the second cycle. Pre-test and post-test were conducted both in the pilot experiment and teaching experiment. This study involved 6 students in the pilot experiment who were different

from the students in the teaching experiment, 31 students in the teaching experiment, and a first grade teacher of SDN 179 Palembang.

The data were collected using video registration, photographs, students' written work, and field notes. In the retrospective analysis, the HLT and students' actual learning process during the teaching experiment were compared. The development of students' strategies in solving subtraction problems can be seen by comparing the result of pre-test and post-test in the teaching experiment.

Hypothetical Learning Trajectory (HLT)

The HLT of this study contains six sequences of activities aimed to reach the goal of the study. For each instructional activity, it will be described the goals, the description of activity, and the conjectures of students' thinking.

The HLT in the teaching experiment is elaborated as follows:

Table 1. Overview of the HLT in learning subtraction

Activity	Goals	Description of Activity	Conjectures of students' thinking
Working with Ginger Candies	- Students are able to understand that subtraction is the converse of addition - Students are able to understand the meaning of subtraction as "taking away something"	- Teacher poses a worksheet that consists of three different contexts of subtraction using the ginger candies. - Two groups of students present their solution.	- Some students will use the ginger candies to represent the situation. - Some students will draw the ginger candies. - Some students will use their fingers.
Working with Grain Bracelets	- Students are able to understand that subtraction is the converse of addition - Students are able to understand the meaning of subtraction as "determining the difference between two numbers"	- Teacher poses a worksheet that consists of three different contexts of subtraction using the grain bracelets. - Two groups of students present their solution.	- Some students will use the grain bracelets to represent the situation. - Some students will draw the grain bracelets. - Some students will use their fingers.
Working with the Beads String	- Students are able to use the beads string as a "model of" the situation in solving subtraction problems - Students are able to	- Teacher lays down a string of 100 beads and provides "ten catcher" to catch ten beads. - Students are asked to solve the previous	- Some students can translate the context into the use of beads string correctly and some others cannot. - Some students only

	make "jumps of 10" in the beads string	problems. - Students also are asked to make "jumps of 10" both jumping forward & jumping backward.	count one by one and some others can count by 10.
Working with the Empty Number Line	- Students are able to use the empty number line as a "model for" their thinking in solving subtraction problems - Students are able to make "jumps of 10" in the empty number line	- Students are asked to solve the previous problems. - Students also are asked to make "jumps of 10". - Teacher demonstrates the shift from the drawing of beads string into the drawing of empty number line.	- Some students can transform the drawing of beads string into the drawing of empty number line correctly and some others cannot. - Some students only count one by one and some others can count by 10.
Working with the Beads String and the Empty Number Line	- Students are able to make a shift from the drawing of beads string into the empty number line - Students are able to apply counting back strategy (direct subtraction) and counting on strategy (indirect addition) in solving subtraction problems - Students are able to make "jumps of 10" in the beads string and in the empty number line	- Teacher poses a worksheet that consists of different subtraction problems with large difference and small difference between minuend & subtrahend. - Students are asked to represent the solution using the drawing of beads string and the empty number line.	- Some students can make a shift from the drawing of beads string into the empty number line and some others cannot. - Some students only apply one strategy and some others can apply both direct subtraction and indirect addition. - Some students can make "jumps of 10" in the beads string and in the empty number line & some others cannot.
Solving Subtraction Problems	- Students are able to use the empty number line in solving subtraction problems - Students are able to apply direct subtraction and indirect addition - Students are able to make "jumps of 10" in the empty number line	- Teacher poses a worksheet that consists of subtraction problems as "taking away something" and "determining the difference between two numbers" in the context format and bare number format.	- Some students still use their fingers and some others already use the empty number line. - Some students still use same strategy and some others already use different strategies with "jumps of 10" or not.

Result and Analysis

In this study, we compared our improved HLT and students' actual learning process during the teaching experiment. We looked to the video and selected some critical

moments. We also collected the written works of the students. We analyzed the lesson to observe what students and teacher do, how the activities work, and how the material contributed to the lesson. We investigated whether the HLT supported students' learning.

Pre-Test

Pre-test is given to know the starting points of the students and what they should learn. Some students already know how to count from any number and how to locate the numbers as a basic skill to work with the beads string and the empty number line.
Pre-test also can give the overview of students' strategies in solving subtraction problems. There are several strategies that students applied to solve the questions from the pre-test. We could find that most of the students used their fingers and the arithmetic rack. Some students used the algorithm of subtracting tens and ones separately and a few students made a drawing to come to the solution. Because of those strategies, the students faced difficulties to get the correct answer.

Activity 1 (Working with Ginger Candies)

The teacher gave worksheet that consists of three questions about addition and subtraction with ginger candies context:

1) Dona has 20 ginger candies. Rani gives to her 6 more candies. How many candies that Dona has right now?
2) Dona has 26 ginger candies. She gives some candies to Andi. She still has 20 ginger candies right now. How many candies that Dona gives to Andi?
3) Dona has 26 ginger candies. She gives 6 of those candies to Budi. How many candies that Dona still has?

All groups could answer the first question correctly (20 + 6 = 26), but they used different strategies. For example, Group Jambu still used their fingers to count the result, while Group Mangga used the ginger candies to find the answer.
For the second question, there are two strategies which are used by the students, 26 - 20 = … and 26 - … = 20. Then, the teacher conducted the class discussion to discuss the students' solutions for the first and second question.

All groups did not find difficulties to answer the third question. They used different strategies like they did before. They were able to write 26 - 6 = 20. They also could understand that the meaning of subtraction is "taking away something" in that context. From activity 1, we can see that most of the students could solve the addition and subtraction problems. They used various strategies in solving the problems. Some students used their fingers, some of them used the ginger candies, other students drew the picture of candies, and even there are some students who were able to do the mental calculation for easy number. Most students also were able to understand that subtraction is the converse of addition implicitly. It is indicated by the students could solve the subtraction problems without counting, only by seeing their relation with the addition problem before. All students did not find difficulties to understand the meaning of subtraction as "taking away something" because the already familiar with it.

Activity 2 (Working with Grain Bracelets)

The teacher gave worksheet that consists of three questions about addition and subtraction with grains bracelet context:

1) Farah is stringing 21 grains to make a bracelet. She needs 7 more grains. How many grains which are needed to make Farah's bracelet?
2) Farah's bracelet consists of 28 grains. Those 7 grains are lost. How many grains the remaining?
3) Farah's first bracelet consists of 28 grains. Farah's second bracelet consists of 21 grains. How many grains the differences between the first and the second bracelet?

The students did not find difficulties to answer the addition problem like in the activity 1. All of them could give the correct answer for the first question (21 + 7 = 28). Most students also could understand that the second question is the subtraction problem (28 - 7 = 21). Each group had different strategies to solve the problems. Some groups needed the grains as a manipulative in counting. Some groups only used their fingers. The others made a drawing of the grains bracelet in their worksheet.

There is a group, Group Nanas, who could see the relation between addition and subtraction. The teacher asked them to explain their solution. They said "Farah has 21

grains and she needs 7 more, so it is 21 + 7." Instead of explaining what they already wrote in the worksheet, they practiced their saying using the grains bracelet. They got 28 as the answer. By looking at the bracelet, they could find the relation between the first and the second question. They said that the second question is the converse of the first, "From the first question we get 21 + 7 = 28. So if Farah has 28 grains and she lost 7 grains, she still has 21 grains in her bracelet."

Most students found difficulties to understand the meaning of "differences". The teacher guided them to put in a row and to compare the first and the second bracelet, like in the figure 1. They got 7 as the differences between those bracelets. After getting the differences, they realized that the context is also a type of subtraction problems. Then, the teacher asked the students to make a drawing that represents the situation and to write the mathematical notation from the context, as shown in the figure 2.

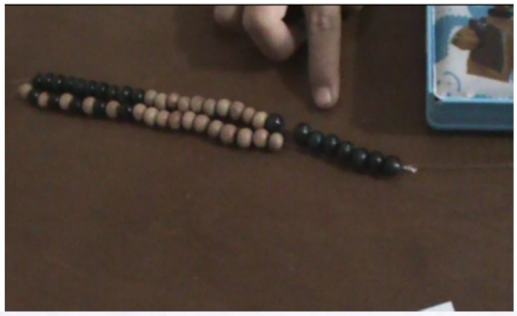

Figure 1. Comparing two grains bracelets

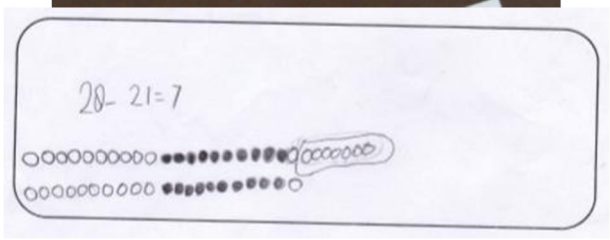

Figure 2. Drawing two grains bracelets

In the end, the teacher emphasized that the meaning of subtraction are not only "taking away something", but also "determining the difference between two numbers".

From activity 2, we can see that most of the students were able to solve the addition and subtraction problems; even they could see the relation between those problems. They used different strategies that more make sense for them. It is quite difficult for the students to construct the other meaning of subtraction. Most of them took a long

time and needed a guidance to understand that subtraction also has the meaning "determining the difference between two numbers". With help of the manipulative, in this case comparing two grains bracelets, the students could see the meaning of "differences" in the subtraction contexts. They also could make the mathematical notation of those contexts.

Activity 3 (Working with the Beads String)

The teacher gave worksheet about two different meanings of subtraction. The contexts in the worksheet are still the same with the last context in the activity 1 and 2. The teacher also prepared small beads string which is contained of 50 beads for each group and one big beads string contained of 100 beads in front of the class. She asked the students to use the beads string as a "model of" the situation in solving subtraction problems. Later on, she held the class discussion to discuss the students' strategies.

The teacher asked Allya (Group Nanas) to count the total beads in the beads string. She found that the beads string consists of ten groups of 10, so the total is 100 beads. Then, she presented her solution in subtracting 26 with 6 from the back. The second problem was solved by Keisya (Group Pisang). She picked up 28 beads and she took away 21 beads from the back one by one like what she did in her group. Because it needed a long time, the teacher asked others who have the easier way to share their strategy. Febi (Group Apel), tried to explain her answer using the beads string in front of the class. She picked up 28 beads and she directly took away 21 beads from the front. She easily got 7 beads as the result of 28 - 21.

After solving the problems using the beads string, the teacher asked the students to draw the representation of the solutions. From the two questions above, there are two groups who could switch their strategy based on the context and the number, Group Nanas and Group Apel. They did subtraction from the back for the first question and subtraction from the front for the second question. Those strategies can be seen in the figure 3 and 4.

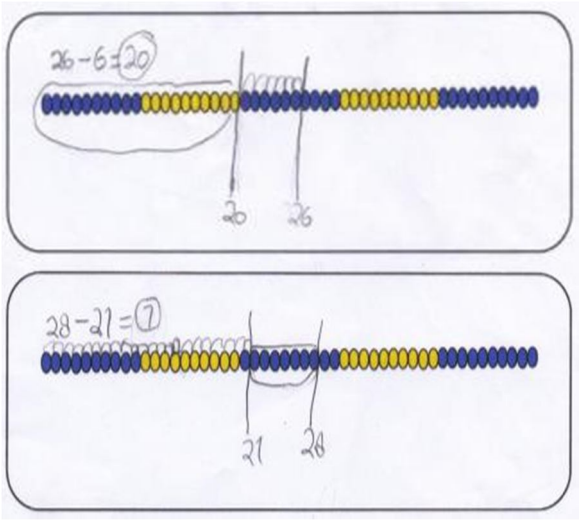

Figure 3. Subtraction from the back Figure 4. Subtraction from the front

In the next session, the teacher guided students to make “jumps of 10” in the beads string, both jumping forward and jumping backward. She used the help of “ten catchers” to catch 10 beads. At the beginning, the teacher jumped from 0 to 10, from 10 to 20, etc, until 90 to 100. She also gave an example by jumping from 100 to 90, from 90 to 80, etc, until 10 to 0. Later on, the teacher challenged students to do jumping forward and backward from different numbers. Most students tended to count the result of jumping before they finally found the pattern of “jumps of 10”.

From activity 3, we can see that most students could recognize the meaning of subtraction as “taking away something” and “determining the difference between two numbers”. The beads string could help the students as a “model of” the situation in solving subtraction problems. When they got confused to solve the word problems, they could represent the situation of the problems in the beads string first. Most of the students could explain with the drawing what they already did in the beads string. There are two strategies which are used by the students in solving subtraction problems, doing subtraction from the back and doing subtraction from the front. Even, some students were able to use those strategies alternately. They could share their strategies in the class discussion so that their friends could choose the easier strategy for the different problems. The beads string and the “ten catchers” were helpful to guide the students in finding the pattern of “jumps of 10”, either jumping forward or jumping backward. Most students were able to do “jumps of 10” without counting anymore.

Activity 4 (Working with the Empty Number Line)

The teacher gave worksheet that consists of the same contexts with worksheet in the activity 3. The difference is in the worksheet 3 she only provided the drawing of the beads string, while in the worksheet 4 she provided the drawing of the beads string and the empty number line in the answer box. Because the students already discussed the solution of the problems, most of the groups applied counting back strategy (direct subtraction) for the first problem and counting on strategy (indirect addition) for the second problem.

The students have similarity in shifting from the drawing of the beads string to the empty number line. They tended to write the number in the empty number line exactly below the position of the bead in the beads string, as seen in the figure 5 and 6. Then, the teacher emphasized that in solving subtraction problems, the students can start from any number and any position in the empty number line.

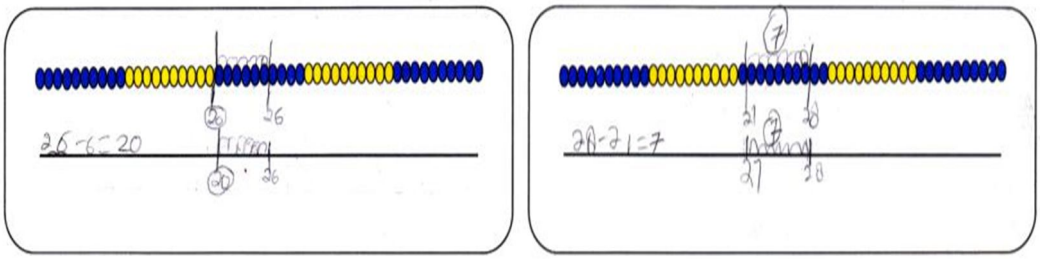

Figure 5. Solving 26-6 in the number line Figure 6. Solving 28-21 in the number line

The teacher gave two more exercises (26 - 3 and 26 - 19) to make students familiar with the two different strategies in solving subtraction problems in the empty number line. She asked the group who did not solve the second exercise by counting back 19 times from 26 to share their strategy. Gisya, from Group Jambu, tried to present her group's strategy. The researcher already guided her to use counting on in solving the second exercise. Surprisingly, Gisya changed her strategy; she did not use counting on anymore. Instead of starting from 19, she started from 26. She counted back until 19 was reached. She counted her small jumps from 26 to 19 and she got 7 jumps. She was able to re-invent the new strategy (indirect subtraction) in the empty number line.

Next, the teacher continued the lesson by posing worksheet that consists of the questions about "jumps of 10" forward and backward from different numbers in the empty number line. Most students already knew that the pattern of jumping backward

is the reverse of jumping forward. Then, the teacher guided them to make the larger jump for "jumps of 10" to differentiate it with jumps one by one.

From activity 4, we can see that most students did not face difficulty in making the shift from the drawing of beads string to the empty number line. The teacher only guided them to change the beads with the line to make the model simpler. Most of them tended to write the number in the empty number line exactly below the position of the bead in the beads string. The students were able to use the empty number line as a "model for" their thinking in solving subtraction problems. They could use the empty number line to solve the different situations with different numbers. The class discussion was very important to make the students realized the possibility in solving subtraction problems with more than one strategy (counting back). Some students could distinguish in applying counting back strategy (direct subtraction) or counting on strategy (indirect addition) based on the problems. Even, there is a student who could re-invent the other strategy by herself, namely indirect subtraction. Almost all students did not have a problem to make "jumps of 10" in the empty number line both jumping forward and jumping backward.

Activity 5 (Working with the Beads String and the Empty Number Line)

The teacher gave worksheet that consists of 6 bare number problems in subtraction, 3 problems with large difference between minuend and subtrahend; and 3 problems with small difference between minuend and subtrahend. Those problems will promote students to use different strategies (direct subtraction and indirect addition). Some of them contain "jumps of 10" problems. Then, the students were asked to represent the problems into the drawing of beads string and the empty number line.

From activity 5, most students were able to represent the solution of the problems in the drawing of beads string and in the empty number line. The students tended to apply direct subtraction (counting back strategy) in solving all of the problems given. By guidance from the teacher, they also could use indirect addition (counting on strategy) to solve certain problems. Like in the previous lesson, there is a student who preferred to use indirect subtraction. In the class discussion, the students recognized that both indirect addition and indirect subtraction are more efficient to solve the problems which have small difference between minuend and subtrahend. Although most of the students did not find difficulty to make "jumps of 10" in the beads string

and in the empty number line, they still could not apply it in solving subtraction problems. They mostly used jumping forward (counting on) or jumping backward (counting back) one by one.

Activity 6 (Solving Subtraction Problems)

The teacher posed worksheet that consists of 6 subtraction problems. Three of them are contextual problems of subtraction as "taking away something" and "determining the difference between two numbers". Three others are bare number problems with large difference and small difference between minuend and subtrahend. There is an empty number line in the answer box to stimulate students to use it in solving the problems.

From activity 6, most of the students were able to use the empty number line as a model in describing their idea about the problems and in representing their solution to solve the problems. By representing students' strategies in the empty number line, each step in students' thinking could be recorded. Therefore, it allowed them to track errors. The students did not face difficulty to solve the contextual problem in subtraction and to find the meaning of subtraction on it. Some students could apply not only direct subtraction, but also indirect addition and indirect subtraction in solving different subtraction problems. Some others still tended to use only one strategy, mostly direct subtraction strategy first, in solving all problems. The empty number line could help the students to visualize the steps needed in counting to come to the result. After finding the difficulty in counting, the students would realize to switch their strategy into indirect addition or indirect subtraction. Some of the students also were able to make the solution simpler by applying "jumps of 10" in the empty number line.

Post-Test

Post-test is given to know the end points of the students after the teaching experiment and what they have learned. By comparing the result from pre-test and post-test, it can be seen the development of students' strategies; whether they are able to apply different strategies in solving subtraction problems up to 100 using the empty number line.

There are several strategies that students used to solve the questions in the post-test. Most students could apply the empty number line with different strategies (direct subtraction, indirect addition, and indirect subtraction) and they could answer correctly. We still found the students who used the mental calculation for easy numbers, the students who used their fingers, and the students who used the algorithm of subtracting tens and ones separately. We did not allow students to use the arithmetic rack and other tools in helping them to answer the questions.

Conclusion

At first, the students used various strategies to solve subtraction problems. Some students used their fingers, some of them used the drawing, other students used the algorithm, and some students could do the mental calculation for easy number. There were also the students who used the arithmetic rack to calculate the result.

When facing the two digit numbers subtraction problems, the students found difficulty to solve the problems with their previous strategies. They needed the ginger candies and the grains bracelets to help them in solving the contextual problems. Later on, those real objects did not longer exist. The students could not rely on them every time in solving the problems.

Then, the students were facilitated to use a model. They were stimulated to shift from situational level to referential level when they have to make representation as the "model of" the situation. A string of beads could serve as a powerful model to represent the situation of those contexts because the students would see the two meanings of subtraction on it. The beads string helped students as a "model of" the situation in solving subtraction problems and it functioned as a bridge in moving from the contextual problems to the use of the empty number line.

In the further activity, the beads string was changed with the model which is simpler and can be applied in the general level. The empty number line served as a "model for" students' thinking in solving the different situation of subtraction independently from a specific situation. Most students still tended to use only one strategy, mostly direct subtraction strategy first, in solving all subtraction problems. The empty number line was helpful to make the students recognized the possibility in solving subtraction with more than one strategy. It could visualize the steps needed in counting to come to the result. After finding the difficulty in counting using direct

subtraction, the students would realize to switch their strategy into indirect addition or indirect subtraction. In the empty number line, the students are supported to apply different strategies (direct subtraction, indirect addition, and indirect subtraction) that more made sense and more efficient for them.

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