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Mathematical Understanding and Proving Abilities: Experiment With Undergraduate Student By Using Modified Moore Learning Approach

Rippi Maya, Utari Sumarmo

Abstract

This paper reports findings of a post test experimental control group design conducted to investigate the role of modified Moore learning approach on improving students' mathematical understanding and proving abilities. Subject of study were 56 undergradute students of one state university in Bandung, who took advanced abstract algebra course. Instrument of study were a set test of mathematical understanding ability, a set test of mathematical proving ability, and a set of students' opinion scale on modified Moore learning approach. Data were analyzed by using two path ANOVA. The study found that proof construction process was more difficult than mathematical understanding task for all students, and students still posed some difficulties on constructing mathematical proof task. The study also found there were not differences between students' abilities on mathematical understanding and on proving abilities of the both classes, and both abilities were classified as mediocre. However, in modified Moore learning approach class there were more students who got above average grades on mathematical understanding than those of conventional class. Moreover, students performed positive opinion toward modified Moore learning approach. They were active in questioning and solving problems, and in explaining their works in front of class as well, while students of conventional teaching prefered to listen to lecturer's explanation. The study also found that there was no interaction between learning approach and students' prior mathematics ability on mathematical understanding and proving abilities, but there were quite strong association between students' mathematical understanding and proving abilities.

Keywords: modified Moore learning approach, mathematical understanding ability, mathematical proving ability.

Abstrak

Artikel ini melaporkan hasil temuan suatu eksperimen berdisain postes kelompok kontrol dengan tujuan mengembangkan kemampuan pemahaman dan pembuktian matematik mahasiswa dengan menerapkan pembelajaran dengan metode Moore termodifikasi. Subyek penelitian sebanyak 56 orang mahasiswa peserta kuliah stuktur aljabar lanjut dari satu universitas negeri di Bandung. Instrumen penelitian terdiri dari tes pemahaman dan tes pembuktian matematik, dan satu set skala pendapat terhadap pembelajaran dengan metode Moore termodifikasi . Analisis data menggunakan anova dua jalur. Penelitian menemukan bahwa tugas pembuktian lebih sukar daripada tugas pemahaman matematik dan terdapat cukup banyak mahasiswa

pada kedua kelas yang mengalami kesulitan pada tugas-tugas pembuktian matematik. Studi juga menemukan tidak ada perbedaan kemampuan mahasiswa dalam pemahaman dan pembuktian matematik pada kedua kelas penelitian, dan kedua kemampuan di atas tergolong sedang. Namun, pada kelas dengan metode Moore termodifikasi terdapat lebih banyak mahasiswa yang memperoleh nilai sedang dan cukup dalam pemahaman matematik daripada pada kelas konvensional. Selain itu mahasiswa menunjukkan sikap positif dan senang terhadap pembelajaran dengan metode Moore termodifikasi, mahasiswa juga aktif bertanya dan menyelesaikan soal-soal pembuktian, serta mampu menjelaskan pekerjaannya di depan kelas, sedang pada kelas konvensional mahasiswa lebih senang mendengarkan penjelasan dosen. Selain itu studi juga menemukan tidak ada interaksi antara pembelajaran dan kemampuan awal terhadap kemampuan pemahaman dan pembuktian matematis, namun terdapat asosiasi yang cukup kuat antara kemampuan pemahaman dan pembuktian matematis mahasiswa.

Kata kunci: metode Moore termodifikasi, pemahaman dan pembuktian matematik, sikap.

Introduction

Some studies reported that mathematical proving was a difficult task for many high school and undergraduate students. Whereas, possessing mathematical proving ability was a certainty ability, because it is an essential ability that should be possess by all students who learn mathematics. Moreover, that ability was needed for pursuing further mathematics contents. That statement was in line with Solow's opinion (1990) that the truth of each proposed mathematical statement must be tested before the statement was used as a basic or reference for testing the truth of other mathematical statement. Afterward, the tested mathematical truth was represented in mathematical language through a proof.

Some researchers (see Arnawa (2006); Barnard (2000); Downs and Downs in Arnawa (2006); Kusnandi (2008); Moore (1994); Moore in Weber (2003); Senk in Hanna and Jahnke (1996); Tall (1999)) conducted studies according to proving abilities. Moore (1994) proposed in detailed seven difficulties on mathematical proving namely:

1) Students did not understand definitions, or they could not state definitions. They consider that definition was an abstract thing.
2) Students had only few intuitive understanding on mathematical concepts.
3) Students' concept images were not enough for carrying out a proof;
4) Students were unable to generate and use their own examples.

5) Students did not know how to use definitions for acquiring structured proof entirely.
6) Students were unable to understand and to use mathematical language and its symbol.
7) Students did not know how to begin proof.

Selden & Selden (1995) cited Moore statements that knowing a definition and could conver example and non example, it didn't mean that the students could master language and logical structure for writing proofs directly. Teaching how to compose a proof was a difficult task. Finding of Senk (in Hanna & Jahnke (1996)) strengthened that statement. Senk reported that from 1520 high school students only 30% students who mastered 75% writing proof in Euclid geometry and only 3% of them who obtained ideal score. Further Tall (1999) strengthened supposition of difficluty to teach proof as well. He stated that proving was an essential mathematical ability, however it was often difficult to teach. Wahyudin (1999) gave similar statement that mathematics teachers only mastered 62,88% mathematics contents and only 50% teachers could prove a rule by using mathematical induction, and only 20 % students mastered mathematics concept correctly. At higher education level, students often say that they can follow a proof that explained by their lecturer in class, but they are unable to compose proofs by themselves when required to do so for homework (Barnard, (2000)). That difficulty was caused by students' unability in investigating mathematical statements deeply.

Furthermore, Moore (Weber, 2003) proposed that sometimes students could state definition of a concept, but they did not understand meaningfully. That disadvantages caused students posed difficulty when they were asked to explain the concept by their own words or to generate an example from a previous concept. Weber (2003), stated that in general students knew what had to do in composing a proof, they could reason deductively, restate and manipulate definition and draw a valid conclussion. However, to know logical rules and definition did not guarantee that students could reason the concept meaningfully. Students should understand a concept intuitively before they could compose a proof, accompanied with habits of active learning in understanding a proof. If students accustomed to cope lecturer's proof but they never composed proof by themselves, then they would pose difficulty to solve mathematical proof. Selden and Selden (Weber, 2003) stated that students' difficulty in proving was caused by

their lack of determination for validating proofs, and they did not know whether a proof was true or false. Moore (Weber, 2003) was sure that students would get a few knowledge of advanced mathematics if they only coped lecturer's proof passively. On the other hand students would learn more mathematics concepts and its proof if they tried to compose a mathematical statement by themselves.

According to Moore's findings about students' difficulties on proving, most of them were caused by their lack of understanding on mathematical concepts and definitions so that they were unable to construct a mathematical proof and to write mathematical notation or to use mathematical language correctly. Therefore, before students able to construct a proof, they should have to master all relevant definitions and theorems. In fact, it is obvious that students' mathematical proving ability associated with their mathematical understanding ability.

The words structure or notation in a proof was standard. Sometimes, it was not easy for some students to understand the proof well, so they might need help to comprehend proof meaningfully. This was happened because some of them considered that proof was only manipulation of unmeaningfull mathematical symbols (Downs & Downs in Arnawa (2006)). Students did not aware that actually proof was really composed by mathematical words and symbols included in a theorem or previous theorems. Researchers observed that students had a little attention on important aspect of mathematical proof, so as long as a theorem could be used for solving mathematical problem, then proof was not a focus of their attention anymore. Besides that, the lack of mathematical understanding used by students did not have ilustration how to begin a mathematical proof. Likewise, mathematics concepts were mutual related to each other. Understanding on a new concept was related by understanding on previous concepts, so it was understandable that without understanding previous concepts students would experience difficulty to explain or even to begin a proof. Arnawa (2006) reported that students, who learned Structure Algebra based on APOS theory, obtained better proving ability than students were taught by conventional teaching.

According to mathematical understanding ability, Polya (Sumarmo, 1987 and 2002) proposed four level of understanding namely, mechanical, inductive, rational, and intuitive understanding. Mechanical understanding happened when a person only memorized rules and implemented it correctly; inductive understanding happened

when he had tried a rule in simple cases and he knew that the rule operated correctly. While rational understanding was obtained when a person knew a rule meaningfully or accompanied with its reason. Furthermore, intuitive understanding was obtained when he was sure on the truth of a rule without doubtful.

Alfeld (2004) stated that a person understood mathematics when he was able to explain mathematics concept in other simpler form of concept, then he was able to connect logically among facts and different concepts; and he could recognize relation between a new concept with previous concepts. When a person mastered all those things, it was called he had good mathematical understanding. According to Polya and Alfeld and some findings that presented before, it was interpreted that in proving a theorem a person should related a new concept or theorem with previous ones. To overcome difficulties on mathematical proving problem, modified Moore method offered learning approach which motivated students to learn mathematical proving actively. Students were motivated to think independently, start with a simple problem for improving a solution accompanied with its supporting reason and to communicate their ideas writtenly or orally so that it could be understood by other students. In written communication students could write it down on a board or in an artickel form, while in oral communication students presented and defended it in front of class.

To consider the characteristics of modified Moore learning approach, it was predicted that the learning approach would train students to have self regulated thinking on solving problem, had abilities of composing relevant reason, and convincing other students through a written or oral presentation. With little lecturer's guidance, this learning approach would able to help students overcame their difficulties on mathematical understanding and proving tasks. In order to obtain optimal result it was suggested that this learning approach was implemented to no more than 24 students. When there were more than 24 students so we could form into groups which the members had various prior mathematics abilities.

Abstract algebra was classified as an advanced and difficult course which contains more proving tasks. Many students failed this course. To overcome this problem, Arnawa (2006) and Nurlaelah (2009) conducted studies which implemented modified APOS theory to improve students' proving ability and mathematical power. Both studies reported that modified APOS learning approach was more successful than conventional teaching in improving students' proving ability and mathematical power.

Those arguments and some findings of studies which implemented modified Moore and modified APOS learning appproaches motivated researcher to conduct an experiment by implementing modified Moore learning approach to improve students' mathematical understanding and proving abilities in advanced abstract algebra course. Considering that mathematics as a systematic science, it was predicted that besides learning approach, students' ability in Structure Algebra would have important role in improving those abilities in advanced structure algebra as well.

Methodolgy

The main goals of this study namely: 1) Were abilities on mathematical understanding and proving abilities of students taught by using modified Moore learning approach better than those abilities of students of conventional class? 2) Were there interaction between learning approach and students' prior mathematics ability on students' mathematical understanding ability and on mathematical proving ability? 3) Was there association between mathematical understanding and proving abilities? 4) What was students' disposition on modified learning approach? 5) What kinds of difficulties did students experience in solving mathematical proving problems?

This study was a post-test experimental control group design as follow.

X O

O

Note:

O : mathematical understanding test and mathematical proving test.

X : modified Moore learning approach

Subject of this study were 56 students from two classes of mathematics department of a state university in Bandung, who took Advanced Structure Algebra course. Instruments of this study were: two tests namely a mathematical understanding essay tests consisted of 6 items and a mathematical proving essay test consisted of 5 items, and a disposition scale Likert model consisted of 16 statements. The both tests were composed by using Arikunto (2002) as a guide, while the disposition scale was modified from Sumarmo (2002). The reliability test were calculated with Cornbah alpha and it were 0,66 and 0,62 for understanding test and proving test respectively, while item validity were between 0,32 and 0,55 and between 0,69 an 0,80 for

understanding test and proving test respectively. Further, learning materials for modified Moore learning approach were modified from Mahavier, May & Parker (2006), Cohen (1982), Chalice (1995), Mahavier (1999) and for the Structure Algebra material was modified from Gallian (2006).

Before experiment was conducted and data were analized, students were classified according to the rule as in Table 1.

Table 1. Classification Rules of Students according to MUA and MPA

Classification	**PMA**	**MUA**	**MPA**
High	$70 \leq PMA \leq 100$	$70 \leq MUA \leq 100$	$70 \leq MPA \leq 100$
Medium	$55 \leq PMA < 70$	$55 \leq MUA < 70$	$55 \leq MPA < 70$
Low	$0 \leq PMA < 55$	$0 \leq MUA < 55$	$0 \leq MPA < 55$

Note:

PMA: prior mathematics ability

MUA: mathematical understanding ability

MPA : mathematical proving ability

Based on the rule in Table 1, there was only a person with high PMA and the rest were classified as medium and low levels of PMA. Afterwards data were analyzed by using two path ANOVA which preceded by certain statistics testing relevant to characteristics of the data

In the following we presented sample of instruments of this study.

1) Sample item of mathematical understanding test

   Observe commutative ring Z_{10} and Z_{12}. Suppose *M* and *N* were maximum ideal of Z_{10} and suppose *P* and *Q* were maximum ideal of Z_{12}.

   Determine those are maximum ideal.

2) Sample item of mathematical proving test

   Suppose *R* was commutative ring with unit element, and suppose *I* was ideal of *R*.

   Prove *R/I* was integral domain if and only if *I* was prime ideal.

Findings and Discussion

1. **Students' Mathematical Understanding and Proving Abilities**

Students' Mathematical Understanding and Proving Abilities according to learning approach and level of students' PMA are presented in Table 2.

a) According to students' PMA, in conventional class there were only two groups namely medium (3) and low level (27) of PMA; while in MLA (*Modified Moore Learning Approach*) class there were three groups namely high level (1) PMA, medium level (6) PMA, and low level (19) PMA. This findings pointed out that students' achievement in structure algebra (namely PMA) were clasified as low or many students failled in this course. Because there was only 1 student with high PMA in MLA class, so we could not compare their abilities on MUA and MPA of for both classes.

Table 2. Students' MUA and MPA according to Learning Approach and Level of PMA

PMA	Mathematical Understanding Ability (MUA)						Mathematical Proving Ability (MPA)					
	MLA			CNV			MLA			CNV		
	n	Mean	sd	n	Mean	sd	N	Mean	sd	n	Mean	sd
Low	19	60,58	11,99	27	67,15	16,14	19	50,53	11,29	27	53,70	14,72
Med	6	76,00	13,18	3	83,33	4,51	6	70,83	12,07	3	68,33	16,07
High	1	100		0			1	80,00		0		
Total	26	65,65	15,19	30	68,77	16,11	26	56,35	14,80	30	55,17	15,23

Note:

Ideal score was 100; *n* : number of subject; PAM: Prior mathematics ability;

MLA: Modified Moore learning approach; CNV: conventional teaching

b) For entirely students, MUA of students in MLA class (65,65) was lower than MUA of students in CNV class (68,77). However according to the result of two path ANOVA in Table 3, there was no difference between students' MUA of MLA class and of CNV class. Similar findings of students' MPA namely there was no difference of students' MPA as well (MPA of MLA class was 56,35 and MPA of CNV class was 55,17).

Table 3. Two Path ANOVA of Students' MUA and Students' MPA according to Learning Approach

Source	Sum of Squares	Df	Mean Square	F	Sig.	Ho
MUA:						
Between groups	134,963	1	134,963	0,548	0,462	Accepted*
Inter groups	13289,251	54	246,097			
Total	13424,214	55				
MPA:						
Between groups	19,377	1	19,377	0,086	0,771	Accepted **
Inter groups	12202,051	54	225,964			
Total	12221,429	55				

Note:

(*) Ho: there was no difference of MUA of MLA class and of CNV class;

(**) Ho: there was no difference of MPA of MLA class and of CNV class;

Table 4. Two Path ANOVA of MUA and MPA of Students with Low PMA

Source	Sum of Squares	Df	Mean Square	F	Sig.	Ho
MUA:						
Between groups	481,265	1	481,265	2,26	0,14	Accepted
Inter groups	9362,039	44	212,774	2	0	*
Total	9843,304	45				
MPA:						
Between groups	112,59	1	112,59	0,62	0,43	Accepted
Inter groups	7924,366	44	180,099	5	3	**
Total	8036,957	45				

Note:

(*) Ho: there was no difference of MUA of MLA class and of CNV class with low PMA;

(**) Ho: there was no difference of MPA of MLA class and of CNV class with low PMA ;

c) Based on level of PMA, on medium and low level of PMA there were no difference of students' MUA of MLA class and of CNV class. Similar findings for students' MPA, there were no difference of students' MPA of MLA class and of CNV class as well (see Table 4 and Table 5.).

Table 5. Two Path ANOVA of MUA and MPA of Students with Medium PMA

Source	Sum of Squares	Df	Mean Square	F	Sig.	Ho
MUA:						
Between groups	107,556	1	107,556	0,82	0,39	Accepted
Inter groups	908,667	7	129,810	9	3	*
Total	1016,222	8				
MPA:						
Between groups	12,5	1	12,5	0,07	0,79	Accepted
Inter groups	1237,5	7	176,786	1	8	**
Total	1250,0	8				

Note:

(*) Ho: there was no difference of MUA of MLA class and of CNV class with medium PMA;

(**) Ho: there was no difference of MPA of MLA class and of CNV class with medium PMA ;

d) Based on classification of students' score on MUA and on MPA in Table 6., it was found that number of students who obtain medium and hig scores on MLA class (73 %) was higher than those number students in CNV class (66,6%). However on students' MPA, number of students who obtained medium and high scores on CNV (43,3%) was higher than those number of students in MLA class. Those findings indicated that MLA was little more effective on achieving students' MUA, while CNV was little more effective on obtaining students' MPA.

Table 6. Classification of Students' MUA and MPA on MLA and CNV Classess

MUA	Mathematical Proving Ability (MPA)							
	High		Medium		Low		Total	
	MLA	CNV	MLA	CNV	MLA	CNV	MLA	CNV
High	5 (19,2%)	4 (13,3%)	2 (7,6%)	5 (16,6%)	3 (11,5%)	9 (30%)	10 (38,4%)	18 (60%)
Medium	0	0	2 (7,6%)	0	7 (26,9%)	2 (6,6%)	9 (34,6%)	2 (6,6%)
Low	0	0	1 (3,8%)	4 (13,3%)	6 (23,1)	6 (20%)	7 (26,9%)	10 (33,3%)
Total	5 (19,2%)	4 (13,3%)	5 (19,2%)	9 (30%)	16 (61,5%)	17 (56,6%)	26 (100%)	30 (100%)

e) By using two path ANOVA on Table 7 and Table 8, it was interpreted there were no interaction between learning approach and level of PMA on students' MUA,

and on students' MPA. The graph of those interaction were ilustrated on Diagram 1 and Diagram 2.

Table 7. Two Path ANOVA between Learnng Approaches and Level of PMA on Students' MUA

Source	Sum of Squares	Df	Mean Square	F	Sig.	H_0
Learning approach (A)	327,78	1	327,78	1,628	0,208	Accepted *
Level of PMA (B)	1694,09	1	1694,09	8,412	0,005	Rejected **
A x B	0,99	1	0,99	0,005	0,94	Accepted***

Note:

(*) Ho: there was no difference of MUA between MLA class and of CNV class

(**) Ho: there was no difference of MUA between medium PMA and low PMA

(***)Ho: there was no interaction between learning approaches (MLA and CNV) and level of PMA on students' MUA

Table 8. Two Path ANOVA between learnng Approaches and Level of PMA on Students' MPA

Source	Sum of Squares	Df	Mean Square	F	Sig.	H_0
Learning approach (A)	0,778	1	0,778	0,004	0,948	Accepted *
Level of PMA (B)	2069,92	1	2069,92	11,52	0,001	Rejected **
A x B	54,66	1	54,66	0,304	0,584	Accepted***

Note:

(*) Ho: there was no difference of MUA between MLA class and of CNV class

(**) Ho: there was no difference of MUA between medium PMA and low PMA

(***)Ho: there was no interaction between learning approaches (MLA and CNV) and level of PMA on students' MPA

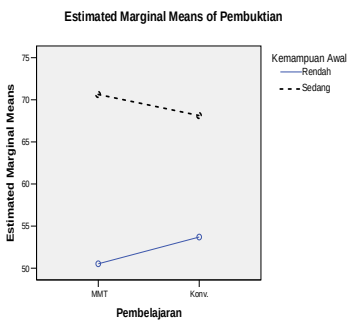

Diagram 1
Interaction between learning approach and level PMA on students' MPA

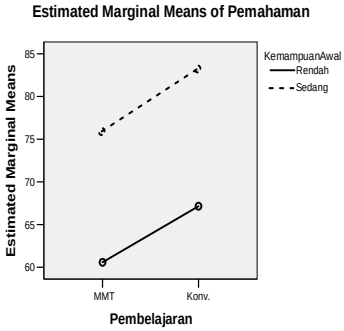

Diagram 2
Interaction between learning approach and level PMA on students' MUA

f) The result from association analysis by using contigency table and statistics χ^2_{cal}, on (Tabel 9.), it were found χ^2_{cal} = 10,8 and χ^2_{tab}, = 9,48 with dk = 4 and $\alpha = 0,05$. Because of $\chi^2_{cal} > \chi^2_{tab}$. so H_0 was rejected. That was meant there was fairly high association between MUA and MPA with coefficient of contigency C = 0,54, from C_{max} = 0,816, or C = 0,66C_{max}.

Table 8. Association between MUA and MPA on MLA Class

MUA	MPA			
	High	Medium	Low	Total
High	5 (19,2%)	2 (7,6%)	3 (11,5%)	10 (38,4%)
Medium	0	2 (7,6%)	7 (26,9%)	9 (34,6%)
Low	0	1 (3,8%)	6 (23,1)	7 (26,9%)
Total	5 (19,2%)	5 (19,2%)	16 (61,5%)	26 (100%)

g) Students disposition on MLA was classified as positive (2,99 of 4). According to its components, students' positive attitude was on disposition on MLA (3,04 of 4); on learning in small group (3,08 of 4), on presentation task (3,09 of 4), while neutral attitude was on learning materials (2,75 of 4). Those findings indicated that learning material needed to revise.

h) Based on result of questioner of some students, it was disclosed that students with medium and high PMA were pleased and could follow MLA learning approach , and stated to obtain higher ability on advanced structure algebra. They liked presentation task as well, but they needed more explanation about exercises they had done and some content in learning materials. In the contrary students with low PMA stated to dislike, were less able to follow the lesson, and obtained less gain on MUA and MPA abilities, exercise tasks were too difficlut, and they needed some explanation about task had to be done. However they felt to get assisstance in small group learning, and liked presentation task.

i) From analysis of students' work, it were found some students' difficulties namely:
   1) Students were not able to generate an example.
   2) Students were not able to explain a concept into simpler form of concepts
   3) Students did not understand standard mathematical notation and mathematical language
   4) Students did not know how to start a proof.
   5) Students' concept of understanding were not enough for starting a proof.
   6) Students' lack of understanding toward mathematical notation caused them used unexact or confusing mathematical language.
   7) Students were not able to seek relation among concepts, definition, theorems, and among theorem and relevant definition.

j) Analysis toward findings of this study compared to previous findings among other things were as follow.
   1) Findings of this study on students'understanding and proving on advanced stucture algebra were lower than Kusnandi's findings (2008) on number theory course, and Nurlaelah's findings on mathematical power on

structure algebra, and were lower than Dasari's findings (2009) on basic statistics. Those were understandable according to some reason such as course of this study was more complex than all previous courses, and this study was conducted during a half semester while the previous study was conducted during one semester. Those argument indicated that for improving advanced abilities such proving task needed longer time and needed to mastered prerequisite contents. Those pointed out that modified Moore learning approach could be implemented better for advanced courses when it was conducted in a longer time and needed to strengthen students understanding on prerequicite courses.

2) The previous studies with modified Moore method were conducted to medium-high students, while this study was conducted to medium-low students. So it was rational that findings of this study was lower than findings of previous studies. However there were similarity findings on students' diffculties between previous studies and this study. Those argument pointed out that modified Moore learning approach could be implemented for various level of students' PMA by revising and adding some explanation and excersices task for students with low level of PMA.

Conclussion and Recommendation

1. Conclusion

Based on findings of this study and discussion, it was obtained some conclusion as follow.

There were no differences on mathematical understanding and proving abilities of students taught by modified Moore learning approach and taught by conventional teaching either entirely or in each level of students' prior mathematics ability. Those abilities were classified as fairly good, however according to number of students who obtained medium and high scores on mathematical understanding ability, in modified Moore learning approach there more students than number of students with medium and high mathematical understanding ability of conventional teaching. Those condition pointed out that modified Moore learning approach was a little more

effective compared to conventional teaching on improving mathematical understanding and proving abilities.

Moreover, during modified learning approach students performed active learning independently, were unafraid to pose question and to present and to explain their ideas in front of class, while students on conventional class were more passive in solving problems and tended to wait lecturer's explanation. Although there were no difference of students' mathematical and proving abilities, but according to learning process, the modified Moore learning approach gave more chances for students to learn actively. Those ilustration supported that modified learning approach was better than conventional teaching in improving mathematical understanding and proving abilities and habits of good learning. The last phrase was very importance for learning further advanced mathematics courses.

Other conclussion of this study was there were no interaction between learning approach and prior mathematics ability on sudents' mathematical understanding and proving abilities. Moreover there was quite strong association between mathematical understanding ability and mathematical proving ability. According to students' opinion on mathematics learning, students of modified Moore learning approach performed positive disposition, namely: they were pleased on modified Moore learning approach, they liked to learn in small group and to present and to explain their work in front of class, and they felt to obtain gain on mathematical understanding and proving abilities on an Advanced Structure Algebra course. However they proposed more explanation about exercises they had done and some contents in learning materials.

Besides those conclussion, there were still some students' difficulties on solving mathematical proof problem namely:

1) students were unable to generate an example.
2) Students were unable to explain a concept into simpler form of concepts
3) Students did not understand standard mathematical notation and mathematical language.
4) Students did not know to start a proof.
5) Students' concept understanding did not satisfy for starting a proof.
6) Students' lack of understanding toward mathematical notation caused they used unexact or confusing mathematical language.

7) Students were unable to seek relation among concepts, definition, theorems, and among theorem and relevant definition.

2. Recommendation

According to discussion of study findings and those conclussion, it were proposed some recommendation as follow.

Considering that mathematical understanding and proving abilities were essensial and difficult tasks and they needed more longer time to learn, so it was recommended that in implementing modified Moore learning approach lecturer should be more patient in giving guidance and presenting excersices task so that students were motivated to compose mathematical proof by themselves, and conduct the lesson in adequate time, namely in one semester. Besides that, lecturer should have cultivated students habits of positive learning disposition continuosly that was needed for learning further advanced mathematics courses.

Previous learning materials of Advanced Structure Algebra should be completed with more examples, ilustration, and excercises with various level of difficulty, and it should be accompanied with relevant guidance and questions that motivated students to learn actively. In implementing the lesson besides students' presentation task, it was also recommended to carried out discussion on students' presentation and some selected excercises tasks in learning material. Part of difficult proof problems that needed more time to solve could be given as home work or group task.

Besides to complete learning material for conducting a similar study on proving ability, it was also recommended to conduct study by implementing modified Moore learning approach for improving other high mathematical thinking such as mathematical critical and creative thinking, communication, reasoning, and problem solving and improving self regulated learning, habits of positive learning such as critical and creative disposition either in abstract algebra or other advance mathematics courses.

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Rippi Maya
State Islamic University - Sunan Gunung Jati, Bandung
Utari Sumarmo
Indonesia University of Education, Bandung